Design Curve for Beams under Impact Loading

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Nomenclature

$\bar{\epsilon}$ $= \epsilon_{max} a^2 / hv$ = generalized strain
a^4 $= EI/\rho A$ = (flexural rigidity)/(linear density)
h = depth of beam
v = impact velocity
L = beam length
m_1 = beam mass
m_2 = impactor mass
M $= m_1/m_2$

Theme

AN exact treatment of the structural impact problem is difficult because of the complicated dynamic behavior and stress field in the contact zone. Even numerical finite-element calculations are tedious and time-consuming. Many simplified approximate analytical models have been proposed in the past, but most are considered inaccurate and of little use to designers. In this paper, we have studied six analytical models for central impact of a simply supported beam, and compared the results with experiments. We have shown that the maximum strain in the beam can be calculated approximately from a simple formula, or a single design curve. For a large impactor mass, the controlling parameter is the ratio of structural mass to impactor mass; the strain is linearly proportional to impact velocity and inversely proportional to the square root of the mass ratio.

Contents

We shall first present the results of six analytical models and some experimental results. Three of the models use lumped mass, the other three involve continuous mass, as shown in Table 1.

In the most elementary of these models, the beam is treated as a massless spring of stiffness $K_1 = 48EI/L^3$, as derived from the static deflection formula. Assuming that all initial kinetic energy of the impactor is converted to potential (strain) energy in the beam, then the maximum beam deflection is

$$w_{1,\,max} = v(m_2/K_1)^{1/2} \tag{1}$$

The additional assumption that the dynamic deflected shape of the beam is identical to that due to a static central point load leads to the following formula for the maximum bending strain.

$$\epsilon_{max} = -\frac{h}{2}\frac{\partial^2 w}{\partial x^2}\bigg|_{x=L/2} = \frac{6h}{L^2} w_{1,max} \tag{2}$$

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Index categories: Structural Dynamic Analysis; Aircraft Structural Design (including Loads).

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Substitution of Eq. (1) into (2) yields

$$\bar{\epsilon} = (3/4M)^{1/2} \tag{3}$$

Note that the generalized strain $\bar{\epsilon}$ is a function of the mass ratio M only.

The next model accounts for the mass of the beam. The beam is represented as a system composed of an equivalent mass $(17/35)m_1$, and a spring of stiffness K_1 (Fig. 1a). If the initial collision is perfectly inelastic and the momentum is conserved, then the maximum deflection is

$$w_{1,\,max} = v[(1+17M/35)m_2/K_1]^{1/2} \tag{4}$$

Substitution into Eq. (2) yields

$$\bar{\epsilon} = \left[\frac{3}{4M(1+17M/35)}\right]^{1/2} \tag{5}$$

which is always less than the value predicted by Eq. (3).

A further improved model includes the effect of elastic contact deformation at the impact point. If the contact force is approximated by $F = K_2(w_2 - w_1)$, then the impact problem may be modeled by the two-degrees-of-freedom system shown in Fig. 1b. Note that the spring K_2 resists compression only. Calculations made with various combinations of M and (K_1/K_2) indicate that (K_1/K_2) affects the maximum strain only slightly; thus $\bar{\epsilon}$ is approximately a function of M only. This model also predicts the phenomenon of multiple collisions, which have been observed in experiments in which M has a low value ($M < 1$).

Observe in Fig. 2 that the three preceding lumped-mass models yield quite similar results for the range of low M.

The Clebsch solution,[1] which is the simplest continuous-mass model, assumes that the rigid impactor is attached to the beam at the impact point. An initial velocity equal to the

Table 1 Summary of the beam impact problem

	Contact Force Neglected		Contact force included
	Energy Conserved	Momentum Conserved	
Lumped Parameter	Lumped spring only	Lumped mass and spring	Two-degree-of-freedom
Continuous Mass	Clebsch	McQuillen	Timoshenko

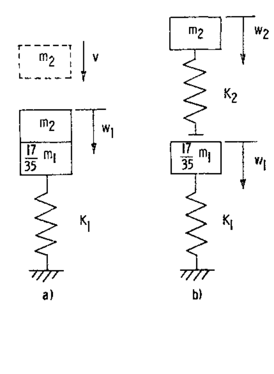

Fig. 1 Lumped parameter models of beam impact. a) One-degree-of-freedom, momentum-conserved model. b) Two-degrees-of-freedom model.

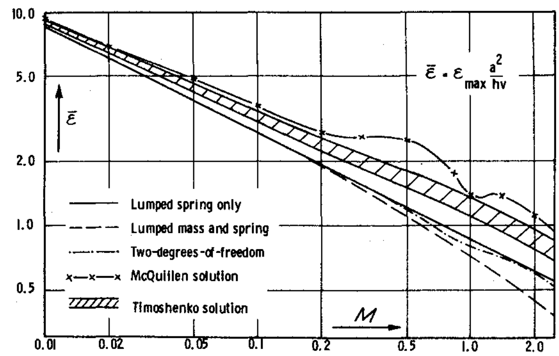

Fig. 2 Generalized strain $\bar{\epsilon}$ vs mass ratio M as calculated by five different methods.

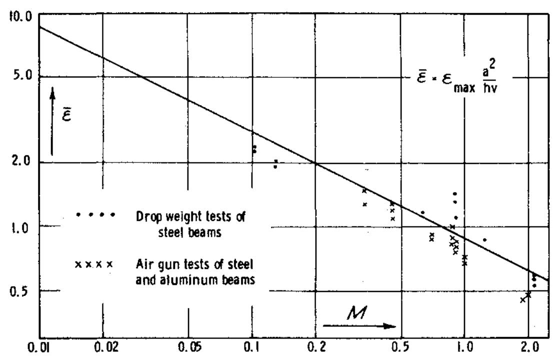

Fig. 3 Data from impact experiments on beams of various dimensions ($L=102$ to 775 mm, $b=15$ to 76 mm, $h=2.3$ to 25.4 mm) and a possible design curve.

original impactor velocity is assumed at the beam point under the impactor, and the subsequent vibration of the beam-impactor system is then derived. The original kinetic energy in the impactor is preserved in this model. The resulting expression for the generalized strain is

$$\bar{\epsilon} = 2 \sum_{i=1}^{\infty} \frac{1}{\phi_i} \frac{\tan\phi_i + \tanh\phi_i}{\sec^2\phi_i - \mathrm{sech}^2\phi_i + 2M/\phi_i^2} \sin\phi_i^2 \tau$$

where the eigenvalues ϕ_i depend only on M; τ is the value of dimensionless time at which the strain reaches its peak. Note that $\bar{\epsilon}$ is again a function solely of M.

The sensitivity of the preceding infinite series to small changes in the time value makes it difficult to find the maximum strain. To facilitate this, McQuillen and Gause[2] have modified the Clebsch model by distributing the impactor mass and initial velocity of the beam, and conserving the momentum. Using the same vibration mode shapes as the Clebsch concentrated-mass system, the coefficients of a truncated series are evaluated using Galerkin's method. Results of this solution are plotted in Fig. 2.

The most sophisticated model we have considered is Timoshenko's solution,[3] which uses continuous mass distribution and includes the contact force. Using Hertz's law of contact, the contact force may be related to the relative displacement between beam and impactor, $F=k_2(w_2-w_1)^{3/2}$. The deflection of the beam w_1 is expressed in terms of the contact force by using a superposition integral derived from the Euler beam equation; the displacement of the impactor w_2 is related to F by Newton's law. The resulting nonlinear integral equation is numerically solved using the computer. Due to this nonlinearity, the beam strain response is a function of three parameters, M, k_2, and v. It can be shown, however, that the dependence of $\bar{\epsilon}$ on k_2 and v is weak; therefore, one single curve of $\bar{\epsilon}$ vs M gives a good approximation of all impact cases.

A series of experiments was conducted in which the strain in the beam was monitored using a resistance-wire strain gage (Micro-Measurements type EA-06-125AD-120) during impact by blunt steel projectiles dropped on or shot at the beam. The results of these experiments are summarized in Fig. 3, in which the curve for the simple energy-conservation model is also included.

An inspection of Figs. 2 and 3 indicates that within 40% of error, the solid line in Fig. 3, or Eq. (3), can be used to estimate the maximum strain. Alternately, any of the more sophisticated models could be used to calculate the $\bar{\epsilon}$ vs M curve. However, without more realistic data on contact energy losses and structural damping, other models may not necessarily give more dependable results. If desired, a few experiments may be conducted to construct an $\bar{\epsilon}$ vs M curve. The key point is that the result may be plotted as a single $\bar{\epsilon}$ vs M curve.

The present study has concentrated on impact of simply supported beams but the same procedure can be easily applied to other structures. Our current work involves applying this approach to clamped and simply supported orthotropic plates.

Acknowledgment

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